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A brief review on the recent laboratory-based experimental proposals to test quantum gravity

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Abstract: Gravitational interaction is distinct from the three other fundamental interactions in nature (namely, electro- magnetic, weak, and strong interactions, which are uncontroversially quantum in nature) in the sense that the status of gravity as a quantum entity is still unsettled. Very recently, few experiments have been proposed to test whether gravity is quantum mechanical. With the current advancement in ground-state cooling of massive objects (with masses $\sim 10^{-14}$ kg), these proposals seem to be feasible with near-term technologies. In particular, manipulating nano-objects is the main requirement to realize these proposals. In this review article, I will briefly describe these experimental proposals based on some concepts of quantum information theory.

Keywords: Quantum entanglement; Superposition; Quantum gravity

1. Introduction

The Einstein field equation depicts that the amount and distribution of mass and energy determine the curvature of space-time and this space-time curvature determines the motion of a mass. Mathematically, this equation can be described as

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \ldots\ldots(1)$$

where $G_{\mu\nu}$ is the Einstein tensor describing the space-time degrees of freedom, and $T_{\mu\nu}$ is the energy-momentum tensor describing the matter degrees of freedom. However, there is an inconsistency in the above equation. Since a mass can be prepared in a spatial superposition state, which is irreducibly quantum, the energy-momentum tensor must be an operator $\hat{T}_{\mu\nu}$ according to quantum mechanics. Hence, the right-hand side of Eq. (1) becomes an operator, which cannot be equal to the scalar $G_{\mu\nu}$.

One possible way to overcome the above problem is to write down $G_{\mu\nu}$ as an operator such that Eq. (1) becomes

$$\hat{G}_{\mu\nu} = \frac{8\pi G}{c^4} \hat{T}_{\mu\nu} \ldots\ldots(2)$$

The above equation tells that gravity sources by a mass in a spatial quantum superposition state is quantum. This leads to the quantum theories of gravity. Various quantum theories of gravity have been proposed till date, e.g., string theory [1], loop quantum gravity [2–4] etc.

Another possibility is to modify Eq. (1) in such a way that the right-hand side becomes a scalar in spite of $\hat{T}_{\mu\nu}$ being an operator. For example, one can take the expectation value of $\hat{T}_{\mu\nu}$ leading to the semiclassical Einstein equation [5-6]:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle \ldots\ldots(3)$$

There are other approaches to modify Eq. (1) such that both sides of this equation remain scalar. All these approaches give rise to classical theories of gravity [5–11] sourced by a mass in spatial quantum superposition.

To date, significant progress has been made in developing quantum theories of gravity or classical theories of gravity sourced by a mass in a quantum state. However, it is still un- settled which of these theories actually describes our nature. In fact, testing the validity of most of these theories requires very high amount of energies, which is almost impossible with current or near-term technologies.

Very recently, some experimental proposals have been presented to test whether gravity is quantum at all [12–22] this is still an open question in modern physics. Interestingly, these experiments do not require very high amount of energy. Manipulating nano-objects in specific quantum states is the primary requirement for this class of experiments. With the current advancement in trapping and ground-state cooling of nano-objects [23–26], these experiments seem to be very promising to be realized in the near future. A positive result of any of these experiments will not tell which particular theory of gravity is correct. However, it will rule out all classical theories of gravity sourced by quantum matters.

In the present article, I will briefly present some of these experimental proposals and possible techniques to realize these experiments.

2. Detecting Gravity-Induced Entanglement

In 2017, S. Bose *et al.* [12-13] and independently, C. Marletto and V. Vedral [14-15] proposed an experiment to test whether gravity is quantum by detecting gravity-induced entanglement. Before going into the details of this proposal, let me briefly recapitulate the concept of quantum entanglement.

Entanglement [27] is a concept of quantum correlation be- tween two or more subsystems that has no classical analogue. A quantum state of two subsystems $|\Psi\rangle_{AB}$A and B is entangled iff it cannot be written in the following product form:

$$|\Psi\rangle_{AB} = |\psi_1\rangle_A \otimes |\psi_2\rangle_B \ldots.(4)$$

If we prepare the joint state of A and B in such an entangled state, then we have complete knowledge about the whole system AB, but we do not have complete knowledge about the subsystem A or B. It can be shown that entanglement between A and B cannot be created by local operations (performed on A and performed on B) and classical communications (between A and B) [27].

Now, let us focus on the proposal by Bose *et al.* - Marletto - Vedral [12-14]. Here, two masses (M_A and M_B) are considered, each of which is prepared in a spatial superposition state.

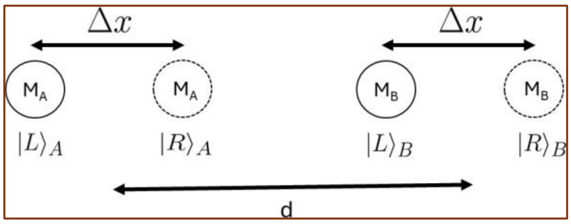

Figure 1: Two test masses M_A and M_B held adjacently in superposition of spatially localized states $|L>_i$ and $|R>_{i\_}$ with i=A, B.

The state of the mass M_A can be expressed as

$$|\psi_1\rangle_A = \frac{1}{\sqrt{2}}(|L\rangle_A + |R\rangle_A), \tag{5}$$

and the state of the mass M_B can be expressed as

$$|\psi_2\rangle_B = \frac{1}{\sqrt{2}}(|L\rangle_B + |R\rangle_B). \tag{6}$$

Here, $|L>_i$ and $|R>_i$ are the states of the mass M_i (i=A, B) located on the left side and the right side, respectively. The centres of $|L>_i$ and $|R>_i$ are separated by Δx and the distance between the centres of the two spatial superpositions is d. The setup is depicted in Fig.1.

The initial joint state of the two masses is in the following product state:

$$|\Psi\rangle_{AB_i} = \frac{1}{\sqrt{2}}(|L\rangle_A + |R\rangle_A) \otimes \frac{1}{\sqrt{2}}(|L\rangle_B + |R\rangle_B). \tag{7}$$

By experimental design, it is ensured that the dominant interaction between the two masses is gravity.

Now, it is assumed that all interactions in the nature (including gravity) are local, i.e., no interaction in nature is action-at-a-distance type interaction (the justification for this assumption is analysed in details in [28].

Under this assumption in the aforementioned setup, if entanglement is created between the two masses, then we can conclude that the dominant interaction between the two masses, which is gravity, must be nonclassical in nature. This is because of the earlier mentioned fact that local operations and classical communications cannot create entanglement. Now, since the interaction between the two masses is local (according to the aforementioned assumption), the creation of entanglement between the two masses can only be explained if the communication or interaction between the two masses is nonclassical.

Now, let us estimate the entanglement generated between the two masses in the above setup if gravity is quantum. If gravity is considered quantum, then the governing Hamiltonian will be given by the Newtonian potential, but in the operator form [29]. In other words, the interaction Hamiltonian of two masses M_1 and M_2 interacting through gravity can be expressed as

$$\hat{H} = G\frac{M_1 M_2}{\hat{r}} \quad \ldots\ldots(8)$$

Consequently, in the above-mentioned setup, the final state of the two masses after they interact for a time interval τ is given by (ignoring global phase)

$$|\Psi\rangle_{AB_f} = \frac{1}{\sqrt{2}}\{|L\rangle_A \frac{1}{\sqrt{2}}\left(|L\rangle_B + e^{i\Delta\phi_{LR}}|R\rangle_B\right) + |R\rangle_A \frac{1}{\sqrt{2}}\left(e^{i\Delta\phi_{RL}}|L\rangle_B + |R\rangle_B\right)\}, \quad (9)$$

where,

$$\Delta\phi_{LR} = \frac{GM_AM_B\tau}{\hbar(d+\Delta x)} - \frac{GM_AM_B\tau}{\hbar d}$$

$$\Delta\phi_{RL} = \frac{GM_AM_B\tau}{\hbar(d-\Delta x)} - \frac{GM_AM_B\tau}{\hbar d}$$

Interestingly, the above state is entangled for any $\Delta\phi_{LR} + \Delta\phi_{RL} \neq 2n\pi$ with n being integer.

In the above set-up, entanglement is generated between the spatial degrees of freedom of the two masses. However, witnessing entanglement between spatial degrees of freedom for large masses (the two masses should be large enough such that they can produce strong enough mutual gravitational interaction to ensure generation of entanglement) is really difficult, as it requires measuring the spatial degrees of freedom in at least two spatial basis (which involves constructing ideal two port beam splitters -- almost impossible task for massive objects).

In order to overcome this problem, a clever method was proposed in [13]. In this proposal, each mass has embedded spin and is subjected to Stern-Gerlach interferometry (involving inhomogeneous magnetic field) [30]. In this case, if gravity is quantum, then entanglement will be generated between the spin degrees of freedom of the two masses. Since witnessing spin entanglement is not challenging (requires measuring spin correlations), it can be concluded whether gravity is nonclassical by detecting spin entanglement.

Now, it is to be ensured that the dominant interaction between the two masses is gravity. It can be noted that all electromagnetic interactions, except one - the Casimir-Polder interaction, can be eliminated by adopting various techniques (see, for example [13] for details). Now, if the distance between the two masses is too large, then the gravitational interaction between them will be very weak. On the other hand, if the distance between the two masses is too small, then the Casimir-Polder interaction will be large compared to the gravitational interaction. Hence, there should be a minimum allowed distance between the two masses such that the gravitational interaction between them becomes at least one order of magnitude larger than the Casimir-Polder interaction. It has been shown [31] that the minimum such distance (given by $d - \Delta x$) between the two masses is 157 μm (considering each of the masses to be diamond with one NV centre, where electronic spin can be embedded in the mass).

The most remarkable feature of this experimental proposal is that if we take the following parameter regimes: M_1, $M_2 \sim 10^{-14}$kg, $d - \Delta x$ (minimum distance between the two masses) ~200 μm (ensuring that the gravitational interaction is one order of magnitude larger than the Casimir-Polder interaction), $\tau \sim 1$ s, $\Delta x \sim 100\ \mu$m, then enough detectable entanglement will be produced.

Whenever we consider a macroscopic mass involving millions of atoms in practical scenario, the mass is in general in mixed state (i.e., in thermal state) due to the random motions of the constituent atoms. However, the primary requirement for most of the quantum experiment with macroscopic masses is to prepare pure states by ground-state cooling. Interestingly, masses of the order of 10^{-14} kg have already been trapped and cooled down to ground-states using various techniques [23-26] in different contexts. Hence, the initial requirement (i.e., preparing pure states of such masses) has already been achieved. This makes this proposal highly promising to be realized in the near future.

A number of arguments have been presented that connect this category of experiments to the nonclassical characteristics of gravity [32-35]. In particular, it has been shown that if these experiments show a positive result (i.e., if gravity-induced entanglement is detected), then it can be concluded that gravity (or, space-time) obeys quantum superposition principle [36-37].

3. Detecting Quantum Measurement Induced Disturbance for Gravity:

The above-mentioned experimental proposal faces a number of drawbacks and practical challenges. For example,

- If the environmental decoherence rate is too high, then no entanglement will be generated [31,38,39]. Hence, this proposal will not work.
- The aforementioned proposal requires witnessing entanglement, for which completely trusted measurement devices are needed. This is really difficult to realize in practical scenarios. One can bypass this problem by witnessing gravity-induced entanglement by performing Bell-type tests [40]. However, this requires even lower decoherence rate (as some mixed entangled states are not Bell nonlocal). In addition, closing all loopholes in a Bell test is really challenging.
- The superposition principle alone cannot uniquely define quantum mechanics. Hence, if any positive result is obtained in the above-mentioned experiment, then it can be concluded that gravity obeys either quantum mechanics, or some other (yet unknown) nonclassical theory supporting superposition principle.

Against this backdrop, another experiment has recently been proposed to test non classicality of gravity [22], which has the following features:

- This proposal will work for any finite decoherence rate [22].
- It will not require any trusted measurement device.
- It can test a different quantum property for gravity. In particular, this proposal aims to test a specific aspect of the quantum measurement postulate, namely the quantum measurement-induced disturbance in the context of gravity. Hence, this test [22], when added together with the earlier-mentioned test [12-14], can take us towards a more complete description of gravity as a quantum entity.

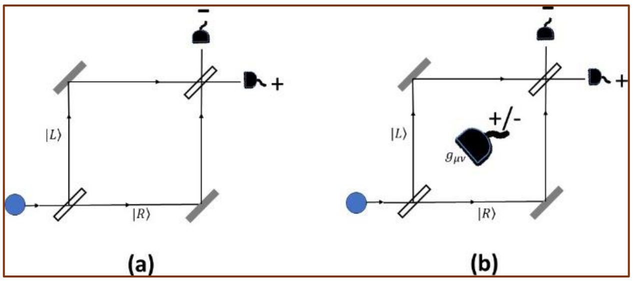

Figure 2: The schematic idea of the proposal presented in [22].

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One aspect of classical physics is that, in principle, one can always measure a classical system without disturbance [40]. For example, when we observe a football match, our act of observation should not have any effect on the result of the match. This is common in our classical world view. This aspect leads to the testable ``Non-Disturbance Condition" (NDC) for classicality [41-43]: The act of performing an intermediate measurement should not influence the outcome-statistics of a subsequent measurement. Obviously, quantum mechanics does not satisfy this condition due to quantum measurement-induced collapse or disturbance. Hence, observing any violation of the NDC would be a signature of nonclassicality. This forms the basis for the proposal depicted in [22]. Next, let me describe the schematics of this proposal.

The basic idea of the proposal is presented in Figure 2. The experiment consists of two parts. In the first part [presented in Figure 2a, a mass (presented by the blue circle in Figure 2) is subjected to an interferometer that creates the spatial superposition $(|\mathrm{L}\rangle + |\mathrm{R}\rangle)/\sqrt{\{2\}}$,where $|L\rangle$ and $|R\rangle$ are two different localized states of the mass passing through two different arms of the interferometer. At the end of the interferometer, outcome + or - is detected. In the second part of the experiment [presented in Figure 2b], everything remains the same, except we place a detector in the mid-way of the interferometer that performs measurement on the gravitational field of the mass. It is crucial to ensure that the measurement by the intermediate detector in Figure 2b is indeed on the gravitational field of the mass. It is not a direct position measurement of the mass by photon scattering or by other electromagnetic interaction. This can be achieved by suitably designing the experiment (see [22] for details).

Now, if gravity is classical, then according to the aforementioned definition of classicality, the intermediate measurement on gravity can be done without any disturbance. Hence, the final probabilities of getting the outcome + or - in the final measurement without any intermediate measurement in Figure 2a should be equal to that after the intermediate measurement on gravity in Figure 2b. Mathematically, it can be expressed in the following way:

$P_{\pm}(\text{Without Intermediate Measurement}) - P_{\pm}(\text{After Intermediate Measurement}) = 0 \ldots (10)$

This is the NDC to be satisfied by classical gravity.

On the other hand, any violation of Eq. [10] will imply that gravity is nonclassical. Because it will imply that the intermediate measurement on gravitational field causes a disturbance on the field, and consequently, according Eq. [2], the mass distribution is also disturbed. This disturbance is detected in the outcome statistics of the final measurement on the mass.

The actual proposal is a bit more complicated than the schematics presented above (see [22] for details of the actual proposal).

In this proposal, we only have to ensure that the intermediate measurement is on the gravitational field. It does not matter here which particular measurement is being implemented here. Hence, this proposal does not require any trusted measurement device.

Interestingly, this proposal, too, can be implemented with masses $\sim 10^{-14}$ kg [22]. Hence, with the recent advancement in ground-state cooling of nano-objects, this proposal also seems to be feasible in the near future. Note that, for both of the proposal [13,14,22], the initial requirements are the same -- we have to cool down a mass $\sim 10^{-14}$ kg to ground-state, and then to create spatial superposition (possibly using mass with embedded spin subjected to Stern-Gerlach interferometry [30].

4. Conclusion

There are four fundamental forces in nature: gravity, electromagnetism, weak interaction, and strong interaction. Among these, electromagnetism, weak interaction, and strong interaction have been confirmed to be quantum in nature. However, whether gravity itself is a quantum force remains an open question. To address this question, few experiments [12-22] have recently been proposed, which have the potential to be realized with near-term technologies. These experiments are inspired from various subtle concepts of quantum information theory and quantum foundations (e.g., quantum entanglement, quantum measurement-induced collapse etc.). Interestingly, unlike the cases to verify different quantum theories of gravity (string theory, loop quantum gravity etc.), the above-mentioned class of experiments does not require very high amount of energy. Rather, trapping and manipulating nano-scale objects are the primary requirements for these experiments. We strongly hope that these experiments will be realized soon, to give us the first genuine quantum signature of gravity.

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